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NAVAL POSTGRADUATE SCHOOL

Monterey, California

POSITION DETERMINATION WITH
LORAN-C TRIPLETS AND THE HEWLETT-PACKARD
HP-67/97 PROGRAMMABLE CALCULATORS

by

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March 1980

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This report presents an algorithm and HP-67/97 programs for position determination with Loran-C chains. Operational data cards are prepared in advance for Loran-C triplets. Position determination is performed using a single program card and an appropriate operational data card.

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The programs in this report are for use within the Navy, and they are presented without representation or warranty of any kind.

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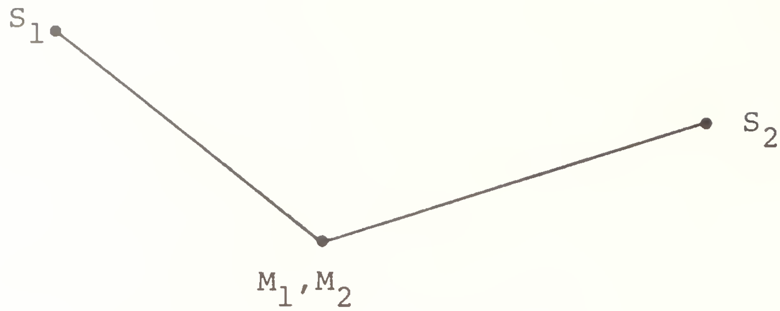
ABSTRACT

This report presents an algorithm and HP-67/97 programs for position determination with Loran-C chains. Operational data cards are prepared in advance for Loran-C triplets. Position determination is performed using a single program card and an appropriate operational data card.

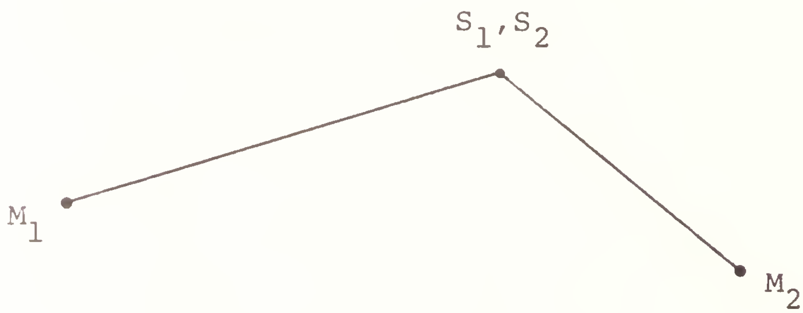
A. Introduction

The Loran system is a radio aid to navigation which utilizes the principle of hyperbolic fixing. The locus of points for which the difference in arrival time of synchronized signals from a pair of transmitters is constant determines a hyperbolic line of positions (LOP). The intersection of two hyperbolic lines of position from two pairs of transmitters determines position or a hyperbolic fix. That two pairs of stations are required for a fix does not necessarily mean that there are four separate stations, for one station of one pair may be colocated with one station of the other pair forming a *Loran triplet* (Figure 1). Triplets may be joined "end-to-end" by station colocation to form a *Loran chain* (Figure 2). Loran chains are common on both the East and West Coasts of the North American continent.

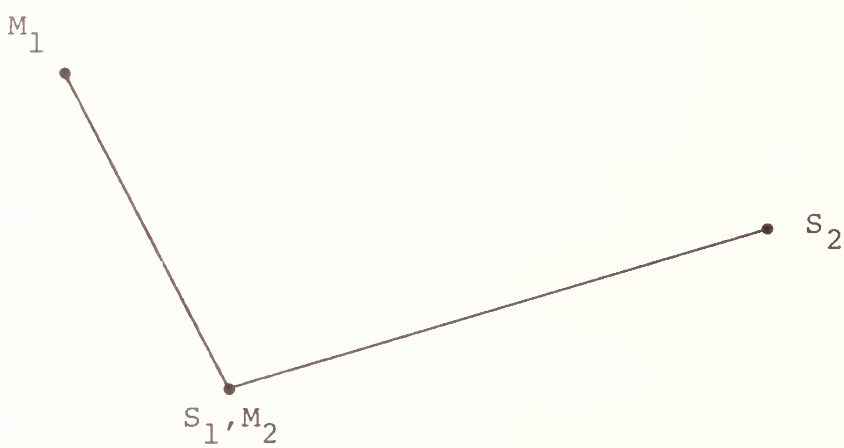
The early "Standard Loran" or Loran-A" operating at a frequency just below 2MHz is still in use in the Pacific area. The present day "Loran-C" operates at 100-kHz and is in use in both the Atlantic and Pacific Areas. The computational algorithm and programs described herein can be used for position determination with Loran-C triplets. Further information on the history, development and operation of the Loran systems may be found in References 1 and 2.



(a) Colocated Master Stations



(b) Colocated Slave Stations



(c) Colocated Master and Slave

Figure 1. Loran Triplets.

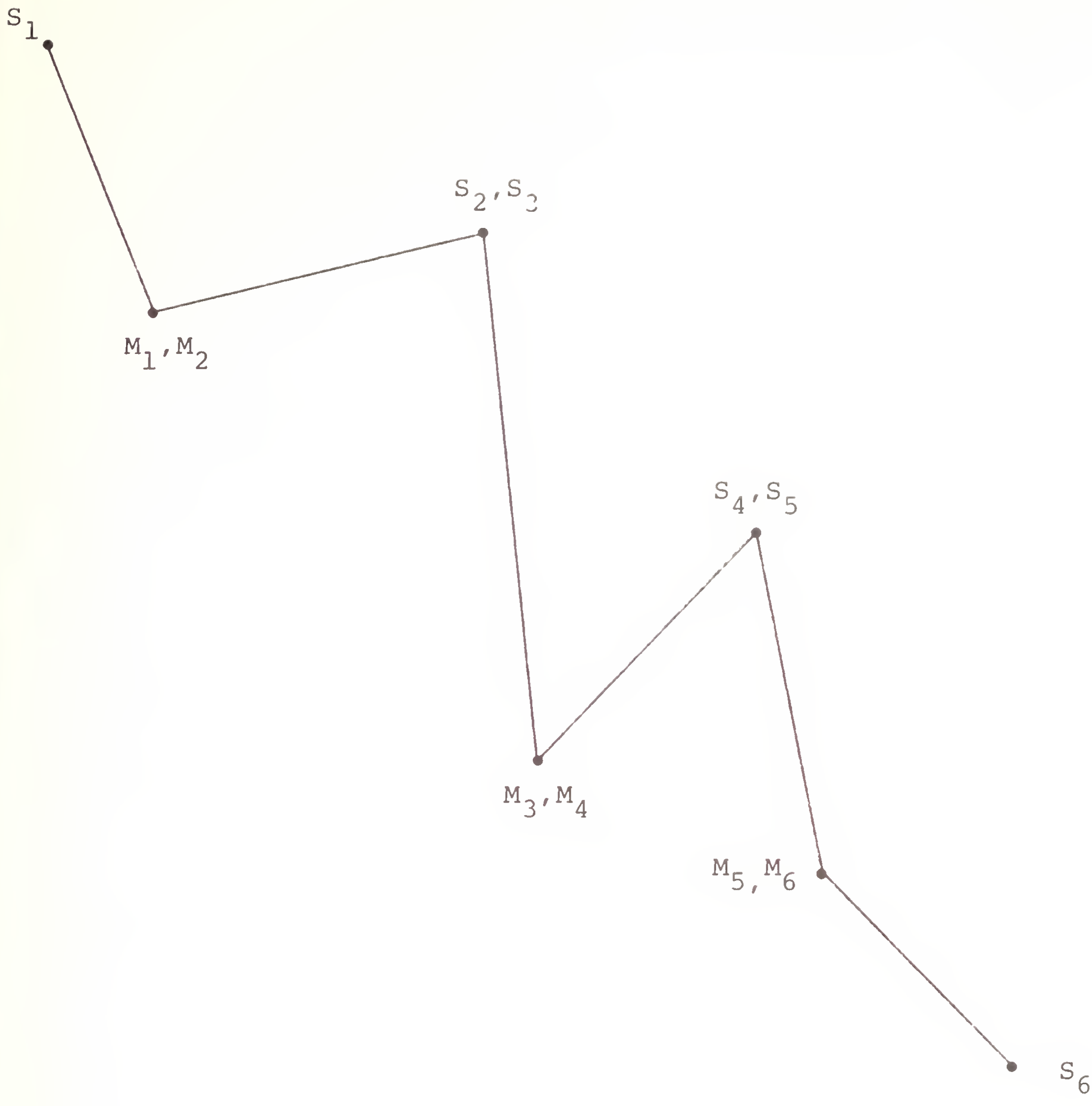


Figure 2. Loran Chain of Five Loran Triplets.

B. Program Description

One program card and one operational data card (described below) are all that is required for on-location position determination from Loran triplet time-difference measurements. Two program cards are required to prepare operational data cards; these operational data cards should be prepared and validated prior to on-location navigational use. Thus although three program cards are described only one program card is required for navigation; two program cards are used to prepare operational data cards during or prior to mission planning. The function of each program card and its intended use follows.

Program Card 1. This program card is used to prepare *master data cards*. A master data card requires the following information for a master(M) station/slave(S) station pair:

1. A M/S pair identification number.
2. The quantity Δt which is the sum of the coding delay plus the one way base line time in microseconds.
3. The latitude and longitude of the master station.
4. The latitude and longitude of the slave station.

Some preprocessing of these data is performed before the master data card is generated. The data generated require only one side of an HP-67/97 magnetic card for each M/S pair, thus a second M/S pair may be placed on side 2 of the card (thus conserving cards) if desired. It is envisaged that a master data card will be prepared in advance for each M/S pair that might be received within an area of operation.

Program Card 2. This program card is used to prepare an *operational data card* for every Loran triplet within an operational area. Each operational data card contains data merged from the master data cards which contain M/S pair information for each pair of the triplet. These merged data are validity checked, colocation of master or slave determined and encoded.

The only inputs required for this program are the two master data cards that comprise the Loran triplet. It is possible to prepare and store operational data cards rather than master data cards. This may be desirable if there is no scarcity of cards and storage space, however the number of possible Loran triplets is considerably larger than the number of M/S pairs.

Program Card 3. This program card is used in conjunction with an operational data card for position determination. Required input is the indicated time difference T for each M/S pair of the triplet. Output is the computed latitude and longitude of the fix. Note: Every Loran fix has two possible solutions. The unwanted solution can almost always be rejected by inspection, however, if the stations of the Loran triplet are nearly aligned then either solution may be valid even though only one solution should be consistent with the flight plan.

C. HP-67/97 Calculator Programs

1. User Instructions

CARD 1

Step	Instructions	Input Data/Units	Keys	Output Data/Units
1.	Read in program card (both sides)			
2.	Input a unique ID number for the Loran pair*	ID	f a	ID
3.	Input the coding delay Δt	Δt	f c	Δt
4a. b.	Input the master station latitude and longitude (CHS for West)**	ϕ_M λ_M	↑ A	--- ---
5a. b.	Input the slave station latitude and longitude (CHS for West)	ϕ_S λ_S	↑ C	--- ---
6a. b.	Run Pass a blank data card through the card reader.	None	E	crd

*Note: Loran pairs are coded on the navigation maps using designators such as 9930X, 9930Y, 9930Z and 9930W. It is suggested that the ID's for these pairs be coded as 9930.1, 9930.2, 9930.3 and 9930.4, respectively. However, any consistent scheme is acceptable.

**The format for position data input is of the form: ±DDD.MMSSFF, where

DDD denotes degrees
MMM denotes minutes
SS denotes seconds
FF dnotes hundredths of a second.

The minus sign (-) denotes Southern latitudes or Western longitudes.

CARD 2

Step	Instructions	Input Data/Units	Keys	Output Data/Units
1.	Read in program card 2			
2a.	Start	---	E	9.00
b.	9.00 will flash in the display. Insert a master data card containing the first pair of a Loran triplet.			
c.	9.00 will flash in the display once more. Insert a master data card containing the second pair of the Loran triplet.			9.00
d.	If the data form a proper triplet, "crd" will appear in the display.			crd
e.	Pass both sides of a blank card thru the card reader to produce the operational data card for the Loran triplet.			

Should "error" appear in the display, then the two master data cards do not compare to form a Loran triplet. Both the latitude and longitude of the colocated stations must be identical on both master data cards in order to successfully produce an operational data card.

Label the A key position with the identification number of the first Loran pair (the pair inserted in Step 2b) and label the B key position with the identification number of the second Loran pair (from Step 2c).

CARD 3

Step	Instructions	Input Data/Units	Keys	Output Data/Units
1.	Read in both sides of the program card 3.			
2.	Read both sides of the operational data card for the Loran triplet that you are receiving.			
3a.	Set to compute Solution A.		f a	---
b.	Set to compute Solution B.		f b	---
4.	Input the observed time delay from the first Loran pair.	T	A	---
5.	Input the observed time delay from the second Loran pair.	T	B	---
6.	Compute fix Latitude Longitude		E R/S	Latitude Longitude
7.	Repeat from Step 2 with a new operational data card or from Steps 3 or 4 as required.			

2. Sample Problem

CARD 1

Step	Instructions	Input Data/Units	Keys	Output Data/Units
	In this series of examples we will prepare and use data cards for the Loran-C pairs 9930X and 9930Y.			
1.	Read in program card 1 (both sides)			
2.	Input the ID for 9930X	9930.1	f a	9930.10
3.	Input the coding delay Δt for 9930X	36389.66	f c	36389.66
4a.	Input the master station latitude	34.034604	↑	---
b.	and longitude (CHS for West)	-77.544676	A	---
5a.	Input the slave station latitude	46.463218	↑	---
b.	and longitude (CHS for West)	-53.102816	C	---
6a.	Compute	None	E	crd
b.	Pass a blank data card through the card reader. Label the card 9930X MASTER			---
7.	Input the ID for 9930Y	9930.2	f a	9930.2
8.	Input the coding delay Δt for 9930Y	52541.31	f c	52541.31
9a.	Input the master station latitude	34.034604	↑	---
b.	and longitude (CHS for West)	-77.544676	A	---
10a.	Input the slave station latitude	41.151193	↑	---
b.	and longitude (CHS for West)	-69.583909	C	---
11a.	Compute	None	E	crd
b.	Pass a blank data card (or the second side of the card used in Step 6b) through the card reader. Label the side 9930Y MASTER			
12.	These two cards will be used in the next example.			

CARD 2

Step	Instructions	Input Data/Units	Keys	Output Data/Units
1.	Read in program card 2.			
2a.	Start	---	E	9.00
b.	While 9.00 is flashing in the display, insert the MASTER data card for station 9930X into the card reader.			
c.	When 9.00 starts flashing in the display again, insert the MASTER data card for station 9930Y into the card reader.	---		9.00
d.	"Crd' will appear in the display.	---		crd
e.	Pass both sides of a blank card through the card reader. Label this card 9930X/9930Y OPERATIONAL DATA CARD. Then label the A key position 9990X and the B key position 9930Y. This card will be used in the next example.	---		0.00

CARD 3

Step	Instructions	Input Data/Units	Keys	Output Data/Units
	You are receiving 9930X and 9930Y and wish to obtain a fix.			
1.	Read in program card 3 (both sides)			---
2.	Read in the operational data card for the triplet 9930X/9930Y (both sides)			---
3.	Select Solution A.		f a	
4.	The indicated time delay is 49400 μs from 9930Y. Input the indicated time delay.	49400	B	0.00
5.	The indicated time delay is 28800 μs from 9930X. Input the indicated time delay.	28800	A	0.00
6.	Solution A: 42°44'57"N Latitude 41°07'32"W Longitude		E R/S	42.4457 -41.0732
	Solution B: 27°00'07"S Latitude 102°27'12"E Longitude Since you are navigating over the North Atlantic, Solution A is the desired fix.			
7.	Repeat from Step 2 with a new operational data card or from Steps 3 or 4 as required.			

3. Program Storage Allocations and Program Listings

Card 1.

Registers:

R0:	ID	S0:	L	RA:	
R1:	Δt	S1:	T	RB:	
R2:	2c	S2:	U	RC:	θ_m
R3:		S3:	V	RD:	$\Delta\theta_m$
R4:	θ_M	S4:	X	RE:	$\Delta\lambda$
R5:	λ_M	S5:	Y	RI:	$\Delta\lambda_m$
R6:	ξ_{MS}	S6:	$\delta_1 d$		
R7:	θ_S	S7:	$\Delta\lambda'_m$		
R8:	λ_S	S8:	d		
R9:	ξ_{SM}	S9:	f		

Initial Flag Status and Use:

0:	OFF, Unused	2:	OFF, Unused
1:	OFF, Unused	3:	OFF, Unused

Display Status:

DSP 4, FIX, DEG.

User Control Keys:

A:	$\phi_M \uparrow \lambda_M$	a:	Station ID
B:		b:	
C:	$\phi_S \uparrow \lambda_S$	c:	Δt
D:		d:	
E:	Prepare data card	e:	

Card 2.

Registers:

R0:	$\pm ID_1$	S0:	$\pm ID_2$	RA:	
R1:	Δt_1	S1:	Δt_2	RB:	
R2:	$2c_1$	S2:	$2c_2$	RC:	
R3:		S3:		RD:	
R4:	θ_{A1}	S4:	θ_{A2}	RE:	$a_p = 21295.87$
R5:	λ_{A1}	S5:	λ_{A2}	RI:	$f = 1/298.26$
R6:	ξ_{A1}	S6:	ξ_{A2}		
R7:	θ_{B1}	S7:	θ_{B2}		
R8:	λ_{B1}	S8:	λ_{B2}		
R9:	ξ_{B1}	S9:	ξ_{B2}		

Initial Flag Status and Use:

0: OFF, Vertex determination

1: OFF, Unused

2: OFF, Validity checking

3: OFF, Unused

Display Status:

DSP 2, FIX, DEG

User Control Keys:

A:		a:	
B:		b:	
C:		c:	
D:		d:	
E:	Run	e:	

Card 3.

Registers:

R0:	ID_1	S0:	ID_2	RA:	M
R1:	Δt_1	S1:	Δt_2	RB:	u, N
R2:	$(2c)_1$	S2:	$(2c)_2$	RC:	D, d
R3:	A_1, c_2, P	S3:	A_2	RD:	$\Delta\sigma$
R4:	$\theta_1 = \theta_F$	S4:	θ_2	RE:	$a_p = 21295.87$
R5:	$\lambda_1 = \lambda_F$	S5:	λ_2	RF:	$f = 1/298.26$
R6:	ξ_1	S6:	ξ_2		
R7:	C_1, c_1, H	S7:	C_2		
R8:	B_1, $S/a = r$	S8:	B_2		
R9:	α_1	S9:			

Initial Flag Status and Use:

0:	OFF, Soln A, Soln B	2:	OFF, M/S Vertex Flag
1:	OFF, Unused	3:	OFF, Unused

Display Status:

DSP 2, FIX, DEG

User Control Keys:

A:	T_1	a:	Soln A
B:	T_2	b:	Soln B
C:		c:	
D:		d:	
E:	Run	e:	

Step	Key	Code	Description
001	*LBLa	21 16 11	Store Station ID.
002	STO0	35 00	
003	RTN	24	
004	*LBLb	21 16 12	Store Δt.
005	STO1	35 01	
006	RTN	24	
007	*LBLA	21 11	Store longitude and store parametric latitude of the master station.
008	HMS→	16 36	
009	STO5	35 05	
010	GSB9	23 09	
011	STO4	35 04	
012	RTN	24	
013	*LBLC	21 13	Store longitude and store parametric latitude of the slave station.
014	HMS→	16 36	
015	STO6	35 06	
016	GSB9	23 09	
017	STO7	35 07	
018	RTN	24	
019	*LBL9	21 09	Subroutine to convert geographic (geodetic) latitude to parametric latitude.
020	X⇄Y	-41	
021	HMS→	16 36	
022	TAN	43	
023	1	01	
024	ENT↑	-21	
025	2	02	
026	9	09	
027	8	08	
028	.	-62	
029	2	02	
030	6	06	
031	1/X	52	
032	P⇄S	16-51	Store flattening constant.
033	STO9	35 09	
034	P⇄S	16-51	
035	-	-45	
036	×	-35	
037	TAN⁻¹	16 43	
038	RTN	24	

Main Routine: Renewal Solution

Step	Key	Code	Compute and/or store:
039	*LBLE	21 15	
040	RCL4	36 04	
041	RCL7	36 07	
042	+	-55	
043	2	02	
044	÷	-24	
045	STOC	35 13	$\theta_m = (\theta_1+\theta_2)/2$
046	RCL7	36 07	
047	RCL4	36 04	
048	-	-45	
049	2	02	
050	÷	-24	
051	STOD	35 14	$\Delta\theta_m = (\theta_2-\theta_1)/2$
052	RCL6	36 06	
053	RCL5	36 05	
054	-	-45	
055	STOE	35 15	$\Delta\lambda = \lambda_2 - \lambda_1$
056	2	02	
057	÷	-24	
058	STOI	35 46	$\Delta\lambda_m = \Delta\lambda/2$
059	P⇄S	16-51	
060	RCLD	36 14	
061	COS	42	
062	X²	53	
063	RCLC	36 13	$H = \cos^2 \Delta\theta_m - \sin^2 \theta_m$
064	SIN	41	
065	X²	53	
066	-	-45	
067	RCLI	36 46	
068	SIN	41	
069	X²	53	
070	×	-35	
071	RCLD	36 14	
072	SIN	41	
073	X²	53	
074	+	-55	
075	STO0	35 00	$L = \sin^2 \Delta\theta_m + H \sin^2 \Delta\lambda_m$
076	ENT↑	-21	

Card 1: Prepare Master Data Card

077	+	-55		115	RCL2	36 02	
078	1	01		116	RCL3	36 03	
079	-	-45		117	-	-45	
080	CHS	-22		118	STO5	35 05	Y = U - V
081	COS⁻¹	16 42		119	CHS	-22	
082	STO8	35 08	$d = \cos^{-1}(1 - 2L)$	120	RCL4	36 04	
083	D→R	16 45		121	RCL1	36 01	
084	LSTX	16-63		122	x	-35	
085	SIN	41		123	+	-55	
086	÷	-24		124	RCL9	36 09	
087	STO1	35 01	T = d/sin d	125	x	-35	
088	RCLC	36 13		126	4	04	
089	SIN	41		127	÷	-24	
090	RCLD	36 14		128	STO6	35 06	$\delta_1 d = f(TX - Y)/4$
091	COS	42		129	CHS	-22	
092	x	-35		130	RCL1	36 01	
093	X²	53		131	+	-55	
094	ENT↑	-21		132	RCL8	36 08	
095	+	-55		133	SIN	41	
096	1	01		134	x	-35	
097	RCL0	36 00		135	R→D	16 46	
098	-	-45		136	P⇄S	16-51	
099	÷	-24		137	STO2	35 02	
100	STO2	35 02	$U = 2 \sin^2 \theta_m \cos^2 \Delta\theta_m/(1-L)$	138	P⇄S	16-51	$2c = S/a_e = (T-\delta_1 d) \sin d$
101	RCLD	36 14		139	RCL5	36 05	
102	SIN	41		140	RCL0	36 00	
103	RCLC	36 13		141	ENT↑	-21	
104	COS	42		142	+	-55	
105	x	-35		143	1	01	
106	X²	53		144	-	-45	
107	ENT↑	-21		145	4	04	
108	+	-55		146	RCL4	36 04	
109	RCL0	36 00		147	-	-45	
110	÷	-24		148	x	-35	
111	STO3	35 03	$V = 2 \sin^2 \Delta\theta_m \cos^2 \theta_m/L$	149	+	-55	F/2 = [Y - (1-2L)(4-X)]
112	RCL2	36 02		150	RCL9	36 09	
113	+	-55		151	RCL1	36 01	
114	STO4	35 04	X = U + V	152	x	-35	-2G = fT

Card 1: Continued.

153	×	-35	
154	CHS	-22	−FG
155	RCLE	36 15	
156	TAN	43	
157	×	-35	
158	4	04	
159	÷	-24	
160	R→D	16 46	$Q = -(FG \tan \Delta\lambda)/4$
161	RCLE	36 15	
162	+	-55	
163	2	02	
164	÷	-24	
165	STO7	35 07	$\Delta\lambda'_m = (\Delta\lambda + Q)/2$
166	1	01	
167	→R	44	
168	RCLD	36 14	
169	COS	42	
170	×	-35	
171	X⇄Y	-41	$\cos \Delta\theta_m \cos \Delta\lambda'_m$
172	RCLC	36 13	
173	SIN	41	$\sin \theta_m \sin \Delta\lambda'_m$
174	×	-35	
175	→P	34	
176	CLX	-51	t_2 is in the Y-register
177	RCL7	36 07	
178	1	01	
179	→P	44	
180	CHS	-22	
181	RCLD	36 14	
182	SIN	41	
183	×	-35	$- \sin \Delta\theta_m \cos \Delta\lambda'_m$
184	X⇄Y	-41	
185	RCLC	36 13	$\cos \theta_m \sin \Delta\lambda'_m$
186	COS	42	
187	×	-35	
188	→P	34	
189	R↓	-31	t_1 is in the X-register
190	CLRG	16-53	

191	P⇄S	16-51	
192	STO6	35 06	Store
193	STO9	35 09	α_{12} in R_6 and
194	R↓	-31	α_{21} in R_9.
195	ST+6	35-55 06	
196	ST-9	35-45 09	
197	0	00	
198	STO3	35 03	
199	WDTA	16-61	
200	R/S	51	

Card 1: Continued.

001	*LBLE	21 15	**Main Routine**
002	CLRG	16-53	
003	P⇄S	16-51	
004	CLRG	16-53	Clear registers.
005	9	09	
006	STOI	35 46	Store 9 in R_I for merging.
007	GSB9	23 09	Input two Master Data Cards.
008	STO3	35 03	
009	P⇄S	16-51	
010	GSB9	23 09	
011	SF0	16 21 00	
012	4	04	
013	STOI	35 46	Flag 0 is used for triplet vertex determination.
014	GSB8	23 08	
015	F2?	16 23 02	Flag 2 is used for validity checking.
016	GTO7	22 07	
017	CF0	16 22 00	Check master stations.
018	7	07	
019	STOI	35 46	
020	GSB8	23 08	Check slave stations.
021	F2?	16 23 02	
022	GTO7	22 07	
023	GSB4	23 04	
024	F2?	16 23 02	Check master/slave colocations for pair 1, and pair 2.
025	GTO5	22 05	
026	GSB4	23 04	
027	F2?	16 23 02	
028	GTO5	22 05	
029	GTO0	22 00	Display 'error' if no colocations found.
030	*LBL7	21 07	
031	F0?	16 23 00	
032	GTO5	22 05	
033	GSB6	23 06	Rearrange data for colocated slave stations.
034	GSB6	23 06	
035	*LBL5	21 05	Continue preparing data card:
036	2	02	
037	1	01	$a_p = \frac{a_e(1+a_2)}{v_0/\eta} = 21295.87$
038	2	02	
039	9	09	
040	5	05	
041	.	-62	
042	8	08	
043	7	07	
044	STOE	35 15	
045	2	02	
046	9	09	
047	8	08	
048	.	-62	
049	2	02	1/f
050	6	06	
051	1/X	52	
052	STOI	35 46	
053	RCL3	36 03	Store two required constants and write Operational Card Data.
054	X=0?	16-43	
055	P⇄S	16-51	
056	WDTA	16-61	
057	CLX	-51	
058	STO3	35 03	
059	R/S	51	
060	*LBL9	21 09	**Subroutine**
061	CF3	16 22 03	Wait for a Master Data Card to be read, then merge and return.
062	MRG	16-62	
063	PSE	16 51	
064	F3?	16 23 03	
065	RTN	24	
066	GTO9	22 09	
067	*LBL8	21 08	**Subroutine**
068	RCLi	36 45	Determine if the pair master stations or slave locations are colocated.
069	P⇄S	16-51	
070	RCLi	36 45	
071	X≠Y?	16-32	
072	RTN	24	
073	ISZI	16 26 46	
074	RCLi	36 45	
075	P⇄S	16-51	
076	RCLi	36 45	Set F2 if a colocation is found.
077	X≠Y?	16-32	
078	RTN	24	
079	SF2	16 21 02	
080	RTN	24	

Card 2: Prepare Operational Data Card.

081	*LBL6	21 06	**Subroutine** Exchange storage of master and slave station data if the slave station is at the vertex of the triplet.
082	RCL7	36 07	
083	RCL4	36 04	
084	STO7	35 07	
085	X⇄Y	-41	
086	STO4	35 04	
087	RCL8	36 08	
088	RCL5	36 05	
089	STO8	35 08	
090	X⇄Y	-41	
091	STO5	35 05	
092	RCL9	36 09	
093	RCL6	36 06	
094	STO9	35 09	
095	X⇄Y	-41	
096	STO6	35 06	
097	RCL0	36 00	Change sign of the ID to signal that the slave station is at the triplet vertex.
098	CHS	-22	
099	STO0	35 00	
100	P⇄S	16-51	
101	RTN	24	
102	*LBL4	21 04	**Subroutine** Determine if the master station of one pair is co-located with the slave station of the other pair.
103	RCL4	36 04	
104	P⇄S	16-51	
105	RCL7	36 07	
106	X≠Y?	16-32	
107	RTN	24	
108	RCL8	36 08	
109	P⇄S	16-51	
110	RCL5	36 05	
111	P⇄S	16-51	
112	X≠Y?	16-32	
113	RTN	24	
114	GSB6	23 06	Rearrange data and set F2 if a colocation is found.
115	SF2	16 21 02	
116	RTN	24	
117	R/S	51	

Card 2: Continued.

001	*LBLc	21 16 13	
002	SF2	16 21 02	Initialize Solution B
003	RTN	24	
004	*LBLa	21 16 11	
005	CF2	16 22 02	Initialize Solution A
006	RTN	24	
007	*LBLB	21 12	
008	P⇄S	16-51	B-Key time delay input
009	GSBA	23 11	
010	P⇄S	16-51	
011	RTN	24	
012	*LBLA	21 11	
013	RCL0	36 00	A-Key time delay input
014	X<0?	16-45	
015	SF2	16 21 02	Set flag 2 if the slave
016	X⇄Y	-41	
017	RCL1	36 01	station is at the vertex.
018	-	-45	
019	RCLE	36 15	
020	÷	-24	
021	R→D	16 46	
022	1	01	
023	→R	44	$2a = (T - \Delta t)/a_p$
024	X⇄Y	-41	
025	F2?	16 23 02	
026	CHS	-22	
027	STO3	35 03	
028	CLX	-51	
029	RCL2	36 02	$A_i = \zeta_i \sin 2a_i$
030	1	01	
031	→R	44	
032	X⇄Y	-41	
033	STO7	35 07	$C_i = \sin 2c_i$
034	R↓	-31	
035	-	-45	
036	STO8	35 08	$B_i = \cos 2a_i - \cos 2c_i$
037	CLX	-51	
038	RTN	24	

039	*LBLE	21 15	Fixing Routine
040	RCL2	36 02	
041	RCL3	36 03	
042	P⇄S	16-51	
043	RCL8	36 08	
044	×	-35	
045	X⇄Y	-41	
046	RCL3	36 03	
047	×	-35	
048	-	-45	
049	STOA	35 11	$\kappa = B_2A_1 - B_1A_2$
050	RCL8	36 08	
051	RCL6	36 06	
052	RCL7	36 07	
053	P⇄S	16-51	
054	RCL8	36 08	
055	×	-35	
056	→P	44	$B_2C_1 \sin \xi_1$
057	X⇄Y	-41	
058	R↑	16-31	$B_2C_1 \cos \xi_1$
059	RCL7	36 07	
060	×	-35	
061	RCL6	36 06	
062	X⇄Y	-41	$B_1C_2 \sin \xi_2$
063	→R	44	
064	R↓	-31	$B_1C_2 \cos \xi_2$
065	-	-45	
066	R↓	-31	
067	X⇄Y	-41	
068	-	-45	C and S
069	R↑	16-31	
070	X⇄Y	-41	ρ and γ
071	→P	34	
072	RCLA	36 11	
073	X⇄Y	-41	
074	÷	-24	Error at Step 75 if time
075	COS⁻¹	16 42	delay input is not valid
076	F0?	16 23 00	for one of the pairs.

Card 3: Loran Fixing Program

077	CHS	-22	$\alpha = \gamma \pm \cos^{-1}(\kappa/\rho)$
078	+	-55	
079	STO9	35 09	Store α
080	RCL6	36 06	
081	-	-45	
082	COS	42	
083	RCL7	36 07	
084	×	-35	
085	RCL3	36 03	
086	+	-55	
087	RCL6	36 06	
088	X⇄Y	-41	
089	→P	34	$r = \text{qatn}\left[\dfrac{B_i}{C_i \cos(\alpha-\xi_i) + A_i}\right]$
090	R↓	-31	
091	STO8	35 08	
092	RCL4	36 04	
093	COS	42	
094	RCL9	36 09	
095	SIN	41	
096	×	-35	
097	STOA	35 11	$M = \cos\theta_1 \sin\alpha_{12}$
098	RCLI	36 46	
099	×	-35	where $\alpha_{12} = \xi_1$
100	STO7	35 07	
101	1	01	$c_1 = fM$
102	RCLA	36 11	
103	X^2	53	
104	-	-45	
105	4	04	
106	÷	-24	
107	RCLI	36 46	
108	×	-35	
109	STO3	35 03	$c_2 = f(1-M^2)/4$
110	2	02	
111	×	-35	
112	CHS	-22	
113	RCL7	36 07	
114	RCLA	36 11	
115	×	-35	
116	-	-45	
117	1	01	
118	+	-55	
119	STOC	35 13	$D = 1 - 2c_2 - c_1 M$
120	1/X	52	
121	ST×3	35-35 03	
122	RCL6	36 06	$P = c_2/D$
123	RCLC	36 13	
124	÷	-24	
125	STOC	35 13	$d = S/(a_e D)$
126	RCL4	36 04	
127	1	01	
128	→F	44	
129	RCL5	36 05	
130	COS	42	
131	×	-35	
132	X⇄Y	-41	
133	→P	34	$\sigma_1 = \text{qatn}(N, \sin\theta_1)$
134	CLX	-51	
135	RCLC	36 13	
136	-	-45	
137	2	02	
138	×	-35	
139	STOB	35 12	$u = 2(\sigma_1 - d)$
140	COS	42	
141	2	02	
142	×	-35	
143	RCL3	36 03	
144	×	-35	
145	CHS	-22	
146	1	01	
147	+	-55	
148	RCLB	36 12	$W = 1 - 2P\cos u$
149	RCLC	36 13	
150	+	-55	
151	COS	42	$V = \cos(u + d)$
152	×	-35	

Card 3: Continued

153	RCL3	36 03	
154	x	-35	
155	2	02	
156	x	-35	
157	RCLC	36 13	
158	SIN	41	Y = 2PVW sin d
159	x	-35	
160	R→D	16 46	
161	CHS	-22	
162	RCLC	36 13	$\Delta\sigma$ = d - Y
163	+	-55	
164	STOD	35 14	
165	ST×7	35-35 07	H = $c_1 \Delta\sigma$
166	RCL4	36 04	
167	COS	42	
168	RCL9	36 09	
169	COS	42	
170	x	-35	N = cos θ_1 cos α_{12}
171	RCLD	36 14	
172	X⇄Y	-41	
173	→R	44	
174	RCLD	36 14	N cos $\Delta\sigma$ and
175	RCL4	36 04	N sin $\Delta\sigma$
176	SIN	41	
177	→R	44	sin θ_1 cos $\Delta\sigma$ and
178	R↑	16-31	
179	+	-55	sin θ_1 cos $\Delta\sigma$
180	X⇄Y	-41	
181	R↑	16-31	sin θ_1 cos $\Delta\sigma$ + N sin $\Delta\sigma$
182	-	-45	
183	X²	53	
184	RCLA	36 11	(N cos $\Delta\sigma$ - sin θ_1 sin $\Delta\sigma)^2$
185	X²	53	
186	+	-55	
187	√X	54	
188	÷	-24	K
189	1	01	
190	RCLI	36 46	

191	-	-45	
192	÷	-24	
193	TAN⁻¹	16 43	
194	→HMS	16 35	
195	PRTX	-14	Display ϕ_2
196	R/S	51	
197	RCL9	36 09	
198	RCLD	36 14	
199	SIN	41	
200	→R	44	sin $\Delta\sigma$ cos α_{12}
201	RCL4	36 04	
202	SIN	41	sin $\Delta\sigma$ sin α_{12}
203	x	-35	
204	CHS	-22	sin θ_1 sin $\Delta\sigma$ cos α_{12}
205	RCL4	36 04	
206	COS	42	
207	RCLD	36 14	
208	COS	42	
209	x	-35	
210	+	-55	cos θ_1 cos $\Delta\sigma$
211	→P	34	- sin θ_1 sin $\Delta\sigma$ cos α_{12}
212	CLX	-51	
213	RCL7	36 07	
214	-	-45	$\Delta\eta$
215	RCL5	36 05	
216	+	-55	$\Delta\lambda = \Delta\eta$ - H
217	1	01	
218	→R	44	$\lambda_2 = \lambda_1 + \Delta\lambda$
219	→P	34	
220	R↓	-31	
221	→HMS	16 35	
222	PRTX	-14	Display λ_2
223	R/S	51	

Card 3: Continued

D. Loran-C Fixing Algorithms

The development of the Loran fixing algorithms in this report is presented in more detail in a companion report [Ref. 3] and will not be repeated here.

The basic Loran-C equation [Ref. 4] can be written as

$$T = [T_S + p(T_S)] - [T_M + p(T_M)] + [T_B + p(T_N)] + \delta \quad (1)$$

where

- T is the "indicated time difference" in microseconds,
- T_M, T_S is the distance, in microseconds, from the master and the slave to the receiver, respectively,
- T_B is the distance, in microseconds, between the master and the slave,
- δ is the assigned coding delay, in microseconds, and
- $p(T)$ is the secondary phase correction, in microseconds, for an all sea water path of length T.

The quantity

$$^{\wedge}t = [T_B + p(T_B)] + \delta$$

is a constant for each master/slave pair. The following World Geodetic System 1972 (WGS 72) values have been adopted for Loran-C navigation [Ref. 4]:

v_0 = 299792458 meters/second is the velocity of light in free space,

η = 1.000338 is the index of refraction of the surface of the earth for standard atmosphere and 100kHz electromagentic waves,

a_e = 6378135.00 meters is the equatorial radius of the earth, and

f = 1/298.26 is the flattening factor ($1 - b/a_e$, where b is the polar radius) of the earth.

Accurate formulas for computing the secondary phase correction $p(T)$ are contained in Reference 4, but for use with the handheld calculator the following linear approximation [Ref. 3] will be used:

$$p(T) = a_1 + a_2 T \ ,$$

where

$$a_1 = -0.321,$$

and

$$a_2 = 0.000635.$$

Using this approximation, it is possible to solve Equation 1 for the quantity $T_S - T_M$. We find

$$T_S - T_M = (T - \Delta t)/(1 + a_2) \ . \qquad (2)$$

On the surface of a sphere a hyperbolic line of position can be represented by the equation [Ref. 3, page 175]

$$\tan r = \frac{\cos 2a - \cos 2c}{\sin 2c \cos \omega + \zeta \sin 2a} \qquad (3)$$

where the origin of the coordinate system is at the prime focus of the spherical hyperbola, $2c$ is the spherical arc joining the foci, $2a$ is a constant for any one LOP, and r and ω are the spherical coordinates of a point on the LOP. If the base line of the coordinate system is the arc joining the foci then ω is the spherical polar angle from the base line to a point P on the LOP and r is the spherical polar distance (or arc) from the prime focus to P. Using the Loran system we take $\zeta = +1$ if the prime focus is at a master station and we take $\zeta = -1$ if the prime focus is at a slave station.

If we take $v = v_0/\eta$ to be the velocity of 100kHz electromagnetic radiation of the earth's surface then

$$2a = v(T_S - T_M)/a_e \ ,$$

or, using Eq. (2),

$$2a = (T - \Delta t)/a_p \ , \qquad (4)$$

where

$$a_p = \frac{a_e(1 + a_2)}{v_0/\eta} = 21295.87 \ \mu s \ .$$

The baseline between master and slave can be obtained from

$$2c = v \ T_B/a_e \ . \qquad (5)$$

Here $2c$ is computed by program card 1 (preparation of master data cards) using the algorithm in Section E.

Consider a Loran-C triplet with master stations colocated. Let ξ_1 and ξ_2 denote the azimuth angles of slave 1 (S_1) and slave 2 (S_2), respectively, measured from North toward the East from the master stations (M) (see Fig. 3). Further, let α and r denote the azimuth and spherical polar arc (distance) of the receiver (R) from M. For this geometry, Eq. (3) can be written as

$$\tan r_i = \frac{B_i}{C_i \cos(\alpha-\xi_i) + A_i} \tag{6}$$

where

$$A_i = \zeta_i \sin 2a_i$$

$$B_i = \cos 2a_i - \cos 2c_i$$

and

$$C_i = \sin 2c_i$$

for the $i^{\underline{th}}$ Loran pair, $i = 1,2$. Since $r = r_1 = r_2$, $\tan r_i$ can be eliminated in Eq. (6). The resulting equation can be rewritten as

$$C \cos \alpha + S \sin \alpha = K , \tag{7}$$

where

$$C = B_1C_2 \cos \xi_2 - B_2C_1 \cos \xi_1,$$

$$S = B_1C_2 \sin \xi_2 - B_2C_1 \sin \xi_1,$$

and

$$\kappa = B_2A_1 - B_1A_2 .$$

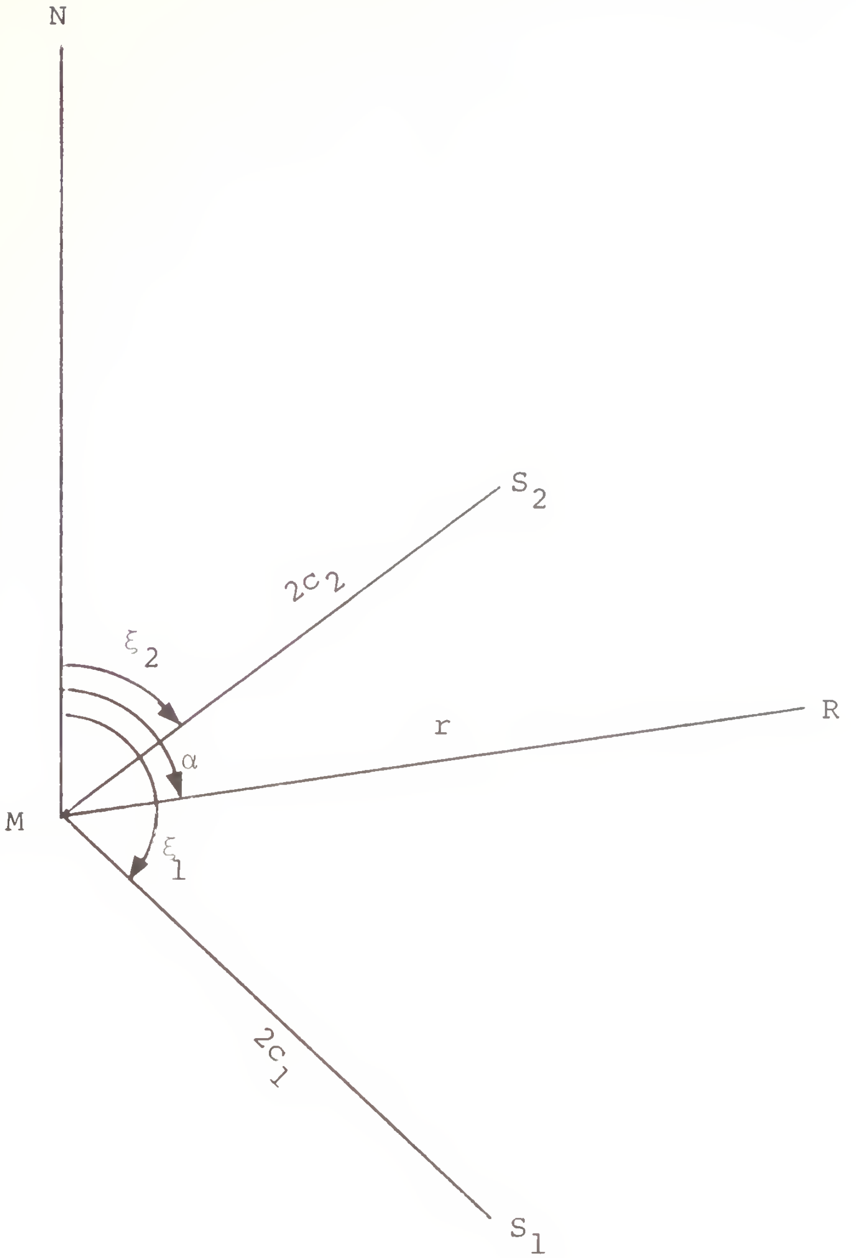


Figure 3. Geometry of a Loran Triplet and a Receiver.

If we define $\rho > 0$ and γ by the equations

$$\rho \cos \gamma = C,$$

and (8)

$$\rho \sin \gamma = S,$$

then

$$\rho = \sqrt{C^2 + S^2} \ ,$$

and

$$\gamma = \mathrm{qatn}(S,C) \ .$$

Here the function qatn(y,x) is the arctangent of y/x adjusted for the proper quadrant according to the signs of x and y. A compact form of this function is

$$\mathrm{qatn}(y,x) = \tan^{-1} \frac{y}{x + 10^{-9} t(x = 0?)} + \pi t(x < 0?)$$

where

$t(z) = 1$ when z is true

and

$t(z) = 0$ when z is false.

When convenient we will use the notation qatn(y/x) interchangeably with qatn(y,x). The qatn function is equivalent to the polar angle obtained using the rectangular to polar conversion function on the HP-67/97.

Now substitute Eq. (8) into Eq. (7) and solve for

$$\alpha = \gamma \pm \cos^{-1}(\kappa/\rho) \tag{9}$$

to obtain the azimuth angle α of the two points of intersection of the LOP's. Finally we obtain a value for r by substituting each α into Eq. (5). We find that

$$r = \text{qatn}\left[\frac{B_i}{C_i \cos(\alpha - \xi_i) + A_i}\right] \quad \text{for} \quad i = 1 \text{ or } 2.$$

The distance and azimuth from M or the triplet vertex can be converted into the latitude and longitude of the two possible positions of R.

The fixing algorithm then uses α and r in the *direct* solution algorithm of spheroidal geodesy (Section F).

E. The Reverse (Inverse) Solution Algorithm

This *reverse* solution algorithm is a modification of the first order in flattening (f) algorithm given by Thomas [Ref. 5, pp. 8-10]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qatn function is defined in Section D. West longitudes (λ) and South latitudes (ϕ) are negative. We are given the points $P_1(\phi_1,\lambda_1)$, $P_2(\phi_2,\lambda_2)$ on the spheroid and are to find the distance S between the points and the forward and back azimuths, α_{12} and α_{21}. Given quantities are ϕ_1, λ_1, ϕ_2 and λ_2. No assumptions about the relative location of P_1 and P_2 are required. The modified *reverse* solution algorithm is:

$$\theta_i = \tan^{-1}[(1-f)\tan\phi_i], \quad i = 1, 2,$$

$$\theta_m = (\theta_1+\theta_2)/2, \quad \Delta\theta_m = (\theta_2-\theta_1)/2, \quad \Delta\lambda = \lambda_2 - \lambda_1,$$

$$\Delta\lambda_m = \Delta\lambda/2, \quad H = \cos^2\Delta\theta_m - \sin^2\theta_m = \cos^2\theta_m - \sin^2\Delta\theta_m = \cos\theta_1\cos\theta_2,$$

$$L = \sin^2\Delta\theta_m + H\sin^2\Delta\lambda_m = \sin^2(d/2), \quad 1 - L = \cos^2(d/2),$$

$$d = \cos^{-1}(1-2L), \quad U = 2\sin^2\theta_m\cos^2\Delta\theta_m/(1-L),$$

$$V = 2\sin^2\Delta\theta_m\cos^2\theta_m/L, \quad X = U + V, \quad Y = U - V,$$

$$T = d/\sin d, \quad \delta_1 d = f(TX - Y)/4, \quad S = a_e(T-\delta_1 d)\sin d,$$

$$F = 2[Y - (1-2L)(4-X)], \quad G = fT/2,$$

$$Q = -(FG\tan\Delta\lambda)/4, \quad \Delta\lambda_m' = (\Delta\lambda + Q)/2$$

$$t_1 = \text{qatn}(-\sin \Delta\theta_m \cos \Delta\lambda'_m, \cos \theta_m \sin \Delta\lambda'_m),$$

$$t_2 = \text{qatn}(\cos \Delta\theta_m \cos \Delta\lambda'_m, \sin \theta_m \sin \Delta\lambda'_m) ,$$

$$\alpha_{12} = t_1 + t_2, \quad \alpha_{21} = t_1 - t_2 .$$

This reverse solution algorithm is used by program card 1 (preparation of master data cards) to compute the baseline distance $2c$ and the azimuths ξ_{MS} and ξ_{SM} between the master and slave stations of a Loran pair.

Details of the modifications made to Thomas' algorithm are contained in Reference 3.

F. The Direct Solution Algorithm

This *direct* solution algorithm is a modification of the first order in flattening (f) algorithm given by Thomas [Ref. 5, pp. 7-8]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qatn function is defined in Section D. West longitudes and South latitudes are negative. We are given the point $P_1(\phi_1, \lambda_1)$ on the spherioid, where ϕ_1, λ_1 are the geodetic latitude and longtiude (geographic coordinates); the forward azimuth α_{12} and the distance S to a second point $P_2(\phi_2, \lambda_2)$; and from these we are to find the geographic coordinates ϕ_2, λ_2 and the back azimuth α_{21}. The given quantities are ϕ_1, λ_1, α_{12} and S. No assumptions about the relative location of P_1 and P_2 are required. The modified *direct* solution algorithm is:

$$\theta_1 = \tan^{-1}[(1-f)\tan\phi_1], \qquad M = \cos\theta_1 \sin\alpha_{12}$$

$$N = \cos\theta_1 \cos\alpha_{12}, \quad c_1 = fM, \quad c_2 = f(1 - M^2)/4,$$

$$D = 1 - 2c_2 - c_1 M, \quad P = c_2/D, \quad \sigma_1 = \mathrm{qatn}(N, \sin\theta_1)$$

$$d = S/(a_e D), \quad u = 2(\sigma_1 - d), \quad W = 1 - 2P\cos u,$$

$$V = \cos(u + d), \quad Y = 2PVW \sin d, \quad \Delta\sigma = d - Y,$$

$$\alpha_{21} = \mathrm{qatn}[-M, -(N\cos\Delta\sigma - \sin\theta_1 \sin\Delta\sigma)] ,$$

$$K = (1-f)[M^2 + (N\cos\Delta\sigma - \sin\theta_1 \sin\Delta\sigma)^2]^{1/2} ,$$

$$\phi_2 = \tan^{-1}[(\sin\theta_1 \cos\Delta\sigma + N\sin\Delta\sigma)/K] ,$$

$$\Delta\eta = \mathrm{qatn}(\sin\Delta\sigma \sin\alpha_{12}, \cos\theta_1 \cos\Delta\sigma - \sin\theta_1 \sin\Delta\sigma \cos\alpha_{12}),$$

$$H = c_1 \Delta\sigma, \quad \Delta\lambda = \Delta\eta - H, \quad \lambda_2 = \lambda_1 + \Delta\lambda.$$

This direct solution algorithm is used by program card 3 (improved fix program) to compute the latitude and longitude of the receiver using the azimuth and range of the receiver from the Loran triplet vertex.

Details of the modifications made to Thomas' algorithm are contained in Reference 3.

G. Discussion and Some Typical Results

The HP-67 program design specifications of COMPATWINGSPAC [Ref. 6] are contained in the following statement.

> "There is a need for an HP-67 program that will compute a geographical position from two Loran delay rate readings. Several methodologies are available to compute the desired position but computational complexities increase with the desired accuracy and flexibility. The most desirable accuracy would be an error of less than 4 n.mi. at a range of 500 n.mi. with less error closer to the stations. It is likely that program length considerations will require that the station pairs have a common site (i.e. two slaves or two masters at the same location). This is not an unusual situation as evidenced by strings of station pairs along coast lines. A data card will probably be necessary for the station pairs to be used. However, more than one program card is unacceptable due to the decrease in functional utility when compared to the manual plotting method. As a final requirement, the fix should be obtainable on either side of the baselines connecting the stations, and not limited to a geometric position relative to one side or the other of the stations."

It was further stated that the maximum computation time to obtain a fix be 1.5 minutes.

It is felt that these design goals have been satisfied. Although one program is required to prepare master data cards for all Loran-C pairs and a second card is required to prepare operational data cards, one each for every triplet, this preparation should be done only once. The data cards should be supplied to users verified and labeled, by the Fleet Mission Program Library. One program card and an appropriate operational data card are all that is required for the fixing algorithm.

The fixing algorithm will display one of the two possible receiver positions in 38 seconds following the entry of the time delay readings. Since there are situations in which *either* of the two solutions could be the valid solution; the decision of which solution to use should be left to the operator, not the program designer.

Testing of the algorithm for all Loran-C triplets and positions relative to those triplets was too extensive a program to be carried out in the available time. Some "typical" scenarios however are presented in Tables I through IV. As can be seen all errors are all well within the design specifications of 4 n.mi at 500 n.mi range from the stations. The time delay values in these Tables were generated using a program discussed in Reference 3. It is recommended that the P-3 community test the algorithm for accuracy in known areas of operation and examine the results for possible regions in which the algorithm may fall outside the design requirements. Such testing should be compatible with the known "unreliable regions" shown on the Loran-C charts.

Table I. Moffett Field South

Position		Indicated Time Delay		Fix		Error
Lat	Long	9940X	9940Y	Lat(N)	Long(W)	n.mi
24°N	122°W	27726.19	40912.76	23°59'55"	122°00'01"	0.08
26	122	27715.97	40998.39	25°59'57"	122°00'01"	0.05
28	122	27702.41	41117.84	27°59'59"	122°00'00"	0.02
30	122	27683.53	41291.85	29°59'59"	122°00'00"	0.02
32	122	27655.47	41555.46	32°00'00"	122"00'00"	0.00
34	122	27609.63	41959.57	34°00'00"	122°00'00"	0.00
36	122	27523.56	42544.11	36°00'00"	121°59'59"	0.01
38	122	27334.61	43248.22	38°00'00"	121°59'58"	0.03

Table II. Moffett Field West

Position		Indicated Time Delay		Fix		Error
Lat	Long	9940Y	9940W	Lat(N)	Long(W)	n.mi
37°N	122°W	42892.86	16257.23	36°59'59"	122°00'01"	0.02
37	125	43056.68	15765.13	37°00'00"	125°00'00"	0.00
37	128	43137.78	15327.12	37°00'00"	128°00'00"	0.00
37	131	43191.10	14970.77	37°00'00"	131°00'00"	0.00
37	134	43232.38	14683.74	37°00'00"	134°00'00"	0.00
37	137	43267.42	14449.40	37°00'00"	137°00'00"	0.00
37	140	43298.80	14254.02	37°00'00"	140°00'01"	0.01
37	143	43327.85	14087.43	37°00'01"	142°59'59"	0.02

Table III. Brunswick Northeast

Position		Indicated Time Delay		Fix		Error
Lat	Long	7930Z	9930X	Lat(N)	Long(W)	n.mi
60°N	30°W	52437.86	28451.72	60°00'03"	29°59'32"	0.24
58	35	51960.93	28391.50	58°00'00"	34°59'46"	0.11
56	40	50992.37	28359.15	55°59'59"	39°59'54"	0.06
54	45	49292.46	28370.85	53°59'59"	44°59'57"	0.03
52	50	47165.60	28490.64	52°00'00"	49°59'59"	0.01
50	55	45236.59	29070.48	50°00'00"	55°00'00"	0.00
48	60	44505.60	30991.94	48°00'00"	60°00'00"	0.00
46	65	44475.70	33697.14	46°00'00"	65°00'00"	0.00
44	70	44588.91	36567.42	43°59'59"	69°59'59"	0.02

Table IV. Jacksonville Southeast

Position		Indicated Time Delay		Fix		Error
Lat	Long	9930W	9930X	Lat(N)	Long(W)	n.mi
9°N	47°W	13058.04	36466.46	8°59'19"	46°59'22"	0.92
12	52	12984.71	37288.35	11°59'34"	51°59'37"	0.57
15	57	12898.73	38267.58	14°59'44"	56°59'47"	0.34
18	62	12793.91	39431.32	17°59'52"	61°59'54"	0.16
21	67	12656.52	40794.36	20°59'56"	66°59'57"	0.08
24	72	12451.30	42330.55	23°59'59"	71°59'59"	0.02
27	77	12097.12	43876.62	27°00'01"	77°00'00"	0.02
30	82	12973.95	44768.53	30°00'01"	82°00'06"	0.09

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2. G. Hefley, *The Development of Loran-C Navigation and Timing*, National Bureau of Standards Monograph 129, U. S. Department of Commerce, U. S. Government Printing Office, Washington, D.C. 20402, October 1972.

3. R. H. Shudde, "An Algorithm for Position Determination Using Loran-C Triplets with a BASIC Program for the Commodore 2001 Microcomputer," Technical Report NPS55-80-009, March 1980, Naval Postgraduate School, Monterey, CA 93940.

4. *LORAN HYPERBOLIC LOP FORMULAS AND GENERAL SPECIFICATIONS FOR LORAN-C* (20 June 1949) were obtained from G. R. Young, Acting Chief, Navigation Department, Defense Mapping Agency, Hydrographic/Topographic Center, Washington, D.C. by private communication, 5 March 1980.

5. Paul D. Thomas, "Spheroidal Geodesics, Reference Systems, and Local Geometry," SP-138, U. S. Naval Oceanographic Office, Washington, D.C., January 1970.

6. Private communication from COMPATWINGSPAC representatives, Moffett Field, CA., October 1979.

APPENDIX. Loran-C Station Parameters

The following list contains the pertinent parameters for each Loran-C station pair. This list was compiled from data in Reference 4. Each column contains the following information:

1. The Loran-C station pair designator
2. Δt, the sum of the coding delay plus one way baseline time, including the secondary phase correction for an all seawater path, in microseconds.
3. The master station latitude.
4. The master station longitude.
5. The slave station latitude.
6. The slave station longitude.

In this list, negative longitudes are West and positive longitudes are East. If desired, this convention may be reversed since the algorithms are independent of such external conventions; if this is done, care should be taken that the signs of all longitudes in the list are reversed. In columns 3 through 6 the latitudes and longitudes appear to be in decimal form, but the actual format is DDD.MMSSFF (which is compatible with the HP-67/97 H.MS input mode) where

DDD designates degrees,
MM designates minutes,
SS designates seconds, and
FF designates hundredths of seconds.

```
4990X,15972.23,16.444395,-169.303120,20.144916,-155.530970
4990Y,34253.18,16.444395,-169.303120,28.234177,-178.173020
5930X,13131.88,46.482720,-067.553771,41.151193,-069.583909
5930Y,28755.02,46.482720,-067.553771,46.463218,-053.102816
5990X,13343.60,51.575878,-122.220224,55.262085,-131.151965
5990Y,28927.36,51.575878,-122.220224,47.034799,-119.443953
5990Z,42266.63,51.575878,-122.220224,50.362972,-127.212935
7930W,15068.02,59.591727,-045.102747,64.542658,-023.552175
7930X,27803.77,59.591727,-045.102747,62.175968,-007.042671
7930Z,48212.20,59.591727,-045.102747,46.463218,-053.102816
7960X,13804.45,63.194281,-142.483190,57.262021,-152.221122
7960Y,29651.14,63.194281,-142.483190,55.262085,-131.151965
7970W,30065.64,62.175968,-007.042671,54.482980,+008.173633
7970X,15048.10,62.175968,-007.042671,68.380615,+014.274700
7970Y,48944.53,62.175968,-007.042671,64.542658,-023.552175
7970Z,63216.30,62.175968,-007.042671,70.545261,-008.435869
7980W,12809.54,30.593874,-085.100930,30.433302,-090.494360
7980X,27443.38,30.593874,-085.100930,26.315501,-097.500009
7980Y 45201.88,30.593874,-085.100930,27.015849,-080.065352
7980Z,61542.72,30.593874,-085.100930,34.034604,-077.544676
7990X,12755.37,38.522061, 016.430596,35.312088, 012.312996
7990Y,31273.30,38.522061, 016.430596,40.582095, 027.520152
7990Z,50999.69,38.522061, 016.430596,42.033649, 003.121590
8970W 14255.11,39.510754,-087.291214,30.593874,-085.100930
8970X 31162.06,39.510754,-087.291214,42.425060,-076.493386
8970Y,47752.74,39.510754,-087.291214,48.364984,-094.331847
9010W 17635.51,34.034604,-077.544676,27.015849,-080.065352
9010X 28289.66,34.034604,-077.544676,46.463218,-053.102816
9010Y,52541.31,34.034604,-077.544676,41.151193,-069.583909
9010Z 63560.72,34.034604,-077.544676,39.510754,-087.291214
9940W,13796.90,39.330662.-118.495637,47.034799,-119.443953
9940X,27894.50,39.330662,-118.495637,38.465699,-122.294453
9940Y,41907.30,39.330662.-118.495637,35.191818,-114.481743
9960W,13797.20,42.425060,-076.493386,46.482720,-067.553771
9960X,26369.93,42.425060,-076.493386,41.151193,-069.583909
9960Y 43221.65,42.425060,-076.493386,34.034604,-077.544676
9960Z 57162.06,42.425060,-076.493386,39.510754,-087.291214
9970W 15283.94,24.48041 , 141.19290 ,24.17077 , 153.58515
9970X,30685.12,24.48041 , 141.19290 ,42.443700, 143.430906
9970Y,59463.13,24.48041 , 141.19290 ,26.362499, 128.085621
9970Z,80746.73,24.48041 , 141.19290 ,09.324566, 138.095523
9990X,14875.32,57.090988,-170.145981,52.494505,+173.105231
9990Y,32069.09,57.090988,-170.145981,65.144012,-166.531447
9990Z,46590.19,57.090988,-170.145981,57.262021,-152.221122
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Coverage of Loran-C Systems

Station	Location
4990	Central Pacific
5930	East Coast, Canada
5990	West Coast, Canada
7930	North Atlantic
7960	Gulf of Alaska
7970	Norwegian Sea
7980	Southeast U.S.A.
7990	Mediterranean Sea
8970	Great Lakes
9930	East Coast, U.S.A.
9940	West Coast, U.S.A.
9960	Northeast U.S.A.
9970	Northwest Pacific
9990	North Pacific

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